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EFFECTS OF A MAGNETIC GUIDE FIELD ON THE PROPAGATION OF INTENSE RELATIVISTIC ELECTRON BEAMS

by

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ABSTRACT

In the absence of a magnetic guide field, intense relativistic electron beams will not propagate because the electrostatic repulsive forces always dominate the magnetic pinching forces. It is necessary to have some background ions to reduce the space charge of the beam. With a sufficiently strong magnetic guide field in the direction of propagation, it is, in principle, possible to have an equilibrium with no ions. The theoretical basis for un-neutralized propagation and beam focusing is presented. Experiments have been carried out that indicate qualitative changes in beam propagation and focusing with a guide field.

I. THEORETICAL DISCUSSION

Electron beams with currents of the order of 10^5 amperes and electron energies of about .5 MeV have been developed and studied in considerable detail at the Laboratory for Plasma Studies[1] and at several other laboratories.[2] For present purposes, an elementary treatment of the electron orbits will suffice. If the beam is propagating in the z-direction and $V_z >> v_x, v_y \; |v_z - V_z|$, the equation of motion for an electron may be approximated as follows

$$\frac{d\underset{\sim}{p}}{dt} = -e \left(\underset{\sim}{E} + \frac{1}{c} \underset{\sim}{v} \times \underset{\sim}{B} \right) \qquad (1)$$

$$\frac{d\underset{\sim}{p}}{dt} = \frac{d}{dt} m\gamma\underset{\sim}{v} = m\gamma \frac{d\underset{\sim}{v}}{dt} + \frac{m\gamma^3}{c^2} \underset{\sim}{v}\,\underset{\sim}{v} \cdot \frac{d\underset{\sim}{v}}{dt}$$

$$\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$$

According to the above approximation[3]

$$\frac{d\underset{\sim}{p}}{dt} \cong m\gamma_o \frac{d\underset{\sim}{v}_\perp}{dt} + m\gamma_o^{\,3} \frac{dv_z}{dt} \qquad (2)$$

where

$$\gamma_o = (1 - V_z^{\,2}/c^2)^{-\frac{1}{2}}$$

$$\underset{\sim}{v}_\perp = \underset{\sim}{e}_x v_x + \underset{\sim}{e}_y v_y$$

Consider a uniform infinitely long beam of radius "a" for which the fields are described by the potentials

$$A_z = Ne\beta_z(r/a)^2 \qquad r \le a \tag{3}$$

$$\Phi = Ne(f-1)\ (r/a)^2 \qquad r \le a$$

$\beta_z = V_z/c$, $N = n\pi a^2$ the line density, where n is the constant electron density. We assume a uniform background ion density and the neutral fraction is $f = n_I/n \le 1$. The equations of motion are

$$\gamma_o m \frac{d^2x}{dt^2} = e \frac{\partial}{\partial x} (\Phi - \beta_z A_z) = -\gamma_o m\Omega^2 x \tag{4}$$

$$\gamma_o m \frac{d^2y}{dt^2} = e \frac{\partial}{\partial y} (\Phi - \beta_z A_z) = -\gamma_o m\Omega^2 y \tag{5}$$

$$\gamma_o{}^3 m \frac{d^2z}{dt^2} = \frac{e}{c} \frac{dA_z}{dt} \tag{6}$$

where

$$\Omega^2 = \frac{2\nu}{\gamma_o} (c/a)^2 (\beta_z{}^2 + f-1)$$

and

$$\nu = Ne^2/mc^2$$

The solutions are elementary and consistent with the assumed approximation if

$$(v_\perp / V_z)^2 \leq (\Omega a / V_z)^2 << 1$$

$$\frac{|v_z - V_z|}{V_z} \leq \frac{\nu}{\gamma_o^3} \beta_z << 1$$

Both inequalities are satisfied if $\nu << \gamma_o$ which can also be written $|I| << I_A$ where $I = -NeV_z$ is the beam current and $I_A = \frac{mc^3}{e} \gamma_o \beta_z = 17000\ \gamma_o \beta_z$ amperes is the Alfvèn critical current. Note that within this approximation Eq. (6) can be replaced by $\gamma_o^3 m \frac{d^2 z}{dt^2} \cong 0$.

If the beam is neutralized f=1, $\Omega^2 > 1$ and the orbits are sinusoidal. In this case, one can expect propagation. There exists a self consistent equilibrium distribution function [4]

$$F(\underset{\sim}{x}, \underset{\sim}{v}) = \frac{\gamma_o m}{2\pi} n\ \delta \left[\frac{\gamma_o m v^2}{2} + U(r) - \varepsilon_o \right] \delta(v_z - V_z) \qquad (7)$$

where

$$U(r) = -e(\Phi - \beta_z A_z) \text{ and } \varepsilon_o = U(a)$$

If the beam is not neutralized, f=o and $\Omega^2 < o$. Then the electron orbits are exponential so that the beam expands on the time scale $|\Omega|^{-1}$ and there exists no self-consistent equilibrium. It has been observed that beams will not propagate in background gases with a pressure less than 50μ which is consistent with this discussion.

We now consider that, in addition to the fields $E_r(r)$, $B_\theta(r)$, there is also a magnetic field $B_z(r)$. Eqs. (4) and (6) are thus modified so that they can be written

$$\frac{d^2x}{dt^2} = \ddot{x} = -\Omega^2 x - \Omega_z \dot{y} = \Omega_z(r)[\, w(r)x - \dot{y}] \tag{8}$$

$$\ddot{y} = -\Omega^2 y + \Omega_z \dot{x} = \Omega_z(r)[\, w(r)y + \dot{x}] \tag{9}$$

Eq. (6) is not affected by the addition of $B_z(r)$.

$$w(r) = \frac{V_z}{r}\frac{B_\theta}{B_z} - \frac{cEr}{rB_z} = -\Omega^2/\Omega_z$$

We first assume B_z= constant and introduce $\zeta = x + iy$ in which case Eqs. (8) and (9) can be combined to give

$$\ddot{\zeta} - i\Omega_z \dot{\zeta} + \Omega^2 \zeta = o \tag{10}$$

The solution is of the form

$$\zeta = A_+ e^{i\omega_+ t} + A_- e^{i\omega_- t}$$

where

$$\omega_\pm = \frac{\Omega_z}{2} \{1 \pm [1 + (2\Omega/\Omega_z)^2]^{\frac{1}{2}}\} \tag{11}$$

We note a qualitative difference in the nature of the solutions. As long as $|(2\Omega)^2/\Omega_z{}^2|<1$ the solutions are always sinusoidal no matter what the sign of Ω^2 is, i.e. even if there is no neutralization and f = o, the solutions are qualitatively the same as when f=1. If $\Omega_z >> 2|\Omega|$, $\omega_+ \cong \Omega_z$ and $\omega_- \cong -\Omega^2/\Omega_z$. The solution of Eq. (10) is approximately

$$\zeta e^{i\frac{\Omega^2}{\Omega_z}t} = r_o e^{i\theta_o} \left[1 + \frac{\Omega^2}{\Omega_z{}^2} e^{i\Omega_z t}\right] \tag{12}$$

This involves a fast gyration of the electron at frequency Ω_z about a guiding center that precesses with frequency $w = -\Omega^2/\Omega_z$. The sign of Ω^2 affects only the direction of the precession of the guiding center. The radius at which the particle moves is approximately r_o and varies only by $r_o(\Omega/\Omega_z)^2$ the gyro-radius for the fast precession. The approximations employed $v_\perp/V_z << 1$ $|v_z - V_z|/V_z << 1$ can be verified a posteriori. The first requires that

$$\frac{v_\perp}{V_z} \simeq \left|\frac{2\Omega^2}{\Omega_z}\frac{a}{V_z}\right| = 2\,\frac{B_\theta(a)}{B_z}\left|1 + \frac{f-1}{\beta_z{}^2}\right| << 1$$

If f=1 $\quad B_z >> 2B_\theta(a)$ $\quad$ is required (13)

If f=o $\quad B_z >> \dfrac{2B_\theta(a)}{\gamma_o{}^2\beta_z{}^2}$ $\quad$ is required.

Since $\gamma_o{}^3 m v_z - \frac{e}{c}A_z(r) = \text{const.}$

$$\left|\frac{v_z - V_z}{V_z}\right| \cong \frac{2\nu}{\gamma_o^{\,3}} \beta_z \frac{\Omega^2}{\Omega_z^{\,2}} = \left|\frac{\beta_z^{\,2} + f - 1}{\gamma_o^{\,2}\beta_z}\right| \left(\frac{B_\theta(a)}{B_z}\right) << 1$$

and the conditions that this inequality is satisfied are similar. It is particularly significant that as long as $B_z >> B_\theta(a)$ the two-mass approximation is applicable no matter what the beam current is.

The solutions of Eqs. (8) and (9) have thus far been discussed only for the case Ω^2=const., Ω_z=const. In view of the fact that the solutions are characterized by a small gyration radius $a_e \cong r_o(\Omega/\Omega_z)^2$, the theory of adiabatic motion can be invoked for $\Omega_z \neq$ const. and $\Omega^2 \neq$ const. provided that changes in these quantities are small over a gyro-radius and during a gyro period, i.e.

$$\left|\frac{a_e}{B} \nabla B\right| << 1 \quad \left|\frac{\Omega_e^{-1}}{B} \frac{\partial B}{\partial t}\right| << 1.$$

Particles spiral along field lines as in Magneto-Hydrodynamic theory and it should be possible to focus such electron beams by increasing the strength of the guide field along the direction of propagation.

In order to find a self-consistent equilibrium that corresponds to a propagating beam, we consider the constants of the motion of Eqs. (8) and (9).

They are

$$\varepsilon = \frac{\gamma_o m}{2}\,(v_x{}^2 + v_y{}^2) - \gamma_o m\int_o^r \Omega_z(r')w(r')r'dr' \tag{14}$$

$$P_\theta = \gamma_o m(xv_y - yv_x) - \gamma_o m\int_o^r \Omega_z(r')r'dr' \tag{15}$$

$$v_z \cong V_z \tag{16}$$

Any function of these constants is an equilibrium distribution function. An appropriate function for present purposes is

$$F(\underset{\sim}{x},\underset{\sim}{v}) = \frac{\gamma_o m}{2\pi}\, n_o\ \delta(\varepsilon - \omega P_\theta)\ \delta(v_z - V_z) \tag{17}$$

$$\varepsilon - \omega P_\theta = \frac{\gamma_o m}{2}\left[(v_x + \omega y)^2 + (v_y - \omega x)^2\right] + U(r)$$

$$U(r) = \gamma_o m\left\{\frac{\omega^2 r^2}{2} + \omega\int_o^r \Omega_z(r')r'dr' + \int_o^r \Omega_z(r')w(r')r'dr'\right\}$$

The moments of $F(\underset{\sim}{x},\underset{\sim}{v})$ required to solve Maxwell's equations are

$$n(r) = \int F(\underset{\sim}{x},\underset{\sim}{v})\,d\underset{\sim}{v}$$

$$= n_o \qquad \text{for } r<a$$

$$= o \qquad \text{for } r>a$$

where "a" is determined from $U(a)=o$

$$n(r)\langle v_z\rangle = n_o V_z \qquad r<a$$

$$= o \qquad r>a$$

$$n(r)\langle v_\theta\rangle = n_o \omega r \qquad r<a$$

$$= o \qquad r>a$$

From these moments we can solve Maxwell's equations

$$\frac{1}{r}\frac{d}{dr} rB_\theta = -\frac{4\pi}{c} n_o e V_z \qquad r\leq a \qquad (18)$$

$$= o \qquad r>a$$

$$\frac{1}{r}\frac{d}{dr} rE_r = -4\pi n_o e \qquad r\leq a \qquad (19)$$

$$= o \qquad r>a$$

$$\frac{dB_z}{dr} = \frac{4\pi e}{c} n_o \omega r \qquad r\leq a \qquad (20)$$

Thus we obtain for E_r and B_θ the usual values for a uniform beam

$$B_\theta = \frac{2I}{cr}\left(\frac{r}{a}\right)^2 \qquad r\leq a \qquad (21)$$

$$= \frac{2I}{cr} \qquad r>a$$

$$E_r = \frac{-2Ne}{r} \left(\frac{r}{a}\right)^2 \qquad r \le a \qquad (22)$$

$$= \frac{-2Ne}{r} \qquad r \ge a$$

where $N = \pi a^2 n_o$, $I = -NeV_z$. From Eq. (22)

$$B_z(r) = B_z(o) + \frac{2Ne}{c} \omega \left(\frac{r}{a}\right)^2 \qquad r \le a \qquad (23)$$

$$= B_z(o) + \frac{2Ne}{c} \omega \qquad r > a$$

Thus

$$U(r) = \left[\frac{e\omega}{c} B_z(o) - \frac{2Ne^2}{\gamma_o^2 a^2} - \gamma_o m\omega^2 \right] \frac{r^2}{2} + \frac{\nu m\omega^2}{a^2} \frac{r^4}{2}$$

The condition $U(a) = o$ defines the beam radius "a" as

$$a = \frac{2}{\gamma_o \beta_z} \frac{|I|/c}{\left\{ B_z(o) \Delta B_z \left[1 + \frac{1}{2} \left(1 - \frac{\gamma_o}{\nu} \right) \frac{\Delta B_z}{B_z(o)} \right] \right\}^{\frac{1}{2}}} \qquad (24)$$

where

$$\Delta B_z = B_z(a) - B_z(o) = \frac{2Ne}{c} \omega$$

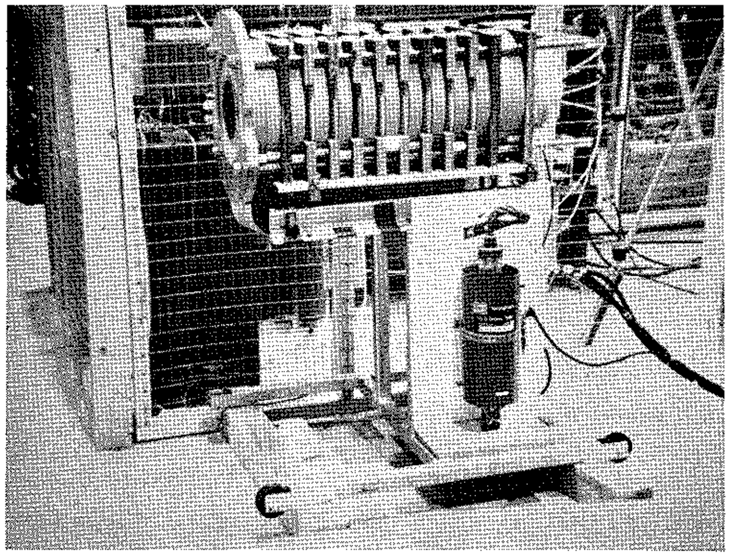
Thus we have obtained a self-consistent equilibrium for a pure electron beam that has a uniform density, is confined to a finite radius and rotates with a constant angular velocity "ω". It is thus plausible that a pure electron beam or an electron beam in a background gas at very low pressure can propagate with a guide field and cannot propagate otherwise. The balance of this paper is devoted to experimental tests of these predictions.

II. EXPERIMENTAL MEASUREMENTS

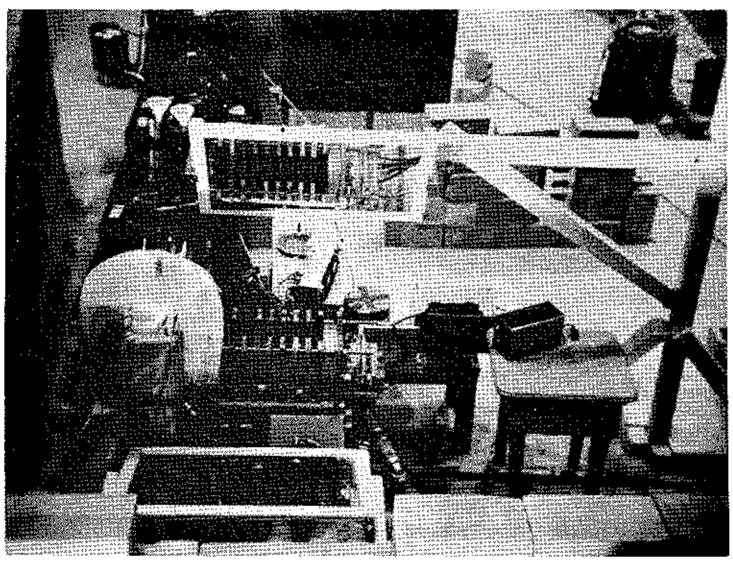
Preliminary studies on two different experimental configurations have been performed. The first system produces the beam in the lower fringing field of a solenoid and then launches the beam into the higher field inside the solenoid. In the second case the beam is created within a uniform axial field and the transport of this beam is examined.

In both configurations the magnetic field is produced by the set of coils shown in Fig. 1(a). There are thirteen identical coils 3 cm thick with a 15 cm inner diameter. They are stacked in a 63 cm array with a 2.5 cm spacing between coils, except at the ends where two coils are placed together to offset the decrease in field near the solenoid ends. This yields a field on axis with less than a one percent ripple from coil to coil and a 53 cm region where the field is constant to within five percent, yet allows room between coils for visual and probe access to the beam environment. The solenoid is fed with a 50 kJ crowbarred capacitor bank, which gives a rise time to peak of 12 msec. and a maximum voltage of 14 kV producing a 20 kG field. A 13.5 cm inner diameter lucite drift tube lined with aluminum screening is mounted within the solenoid.

In the first experiment this solenoid assembly was placed within 6 cm of the anode foil of a field emission diode.[5] This diode is planar with a 10 cm diameter cathode and typically a 1 cm anode-cathode gap. It is fed with a 4 ohm pulse forming line, capable of a 500 keV output. The diode was constructed with .75 cm stainless steel plates and a stainless steel foil holder to provide a minimum reduction of the field in the anode-cathode gap. Thus, the axial field within the diode was forty percent of the maximum field. As shown in Fig. 1(b), the mirrors allow top and side views of the beam through the gaps between coils. Fig. 2 records typical results from this configuration. The top open shutter photograph displays a normal beam propagating without external magnetic field in the drift tube at 0.3 Torr. In the lower photograph the conditions remain the same except that a 10 kG field is applied. The diameter of the beam in the magnetic field is about one-third that of the field free case. A scintillator-diode combination detecting the x-rays produced from an aluminum plate at the end of the field coils recorded one-third the signal with magnetic field compared to no field. Considering the change in beam cross-section, this indicates

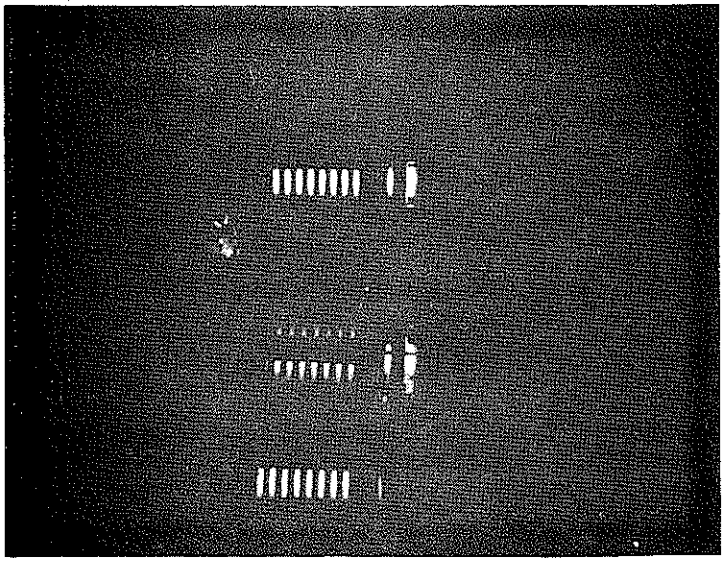


(a.)

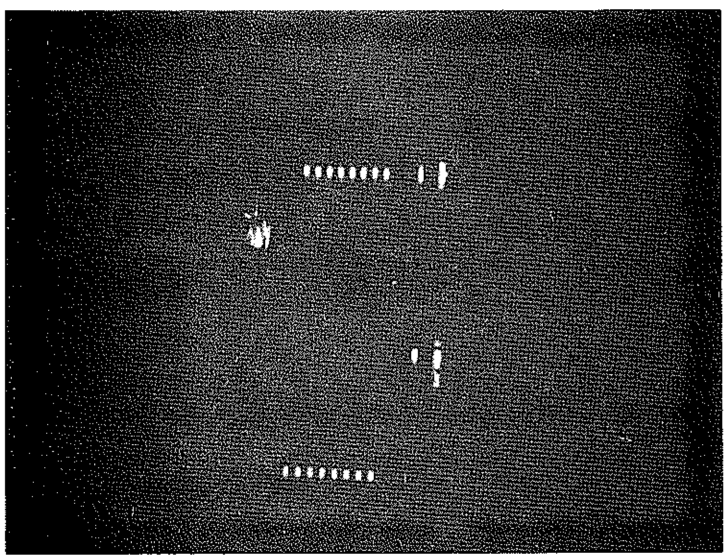


(b.)

Fig. 1 (a.) Photograph of the field coil assembly. (b.) Field coil in place on the pulse line showing the position of the top and side mirrors.



(a.)



(b.)

Fig. 2 (a.) Open shutter photograph of beam light without field using the mirror configuration shown in Fig. 1 (b.). (b.) The same conditions except for the addition of a 10 kG magnetic field.

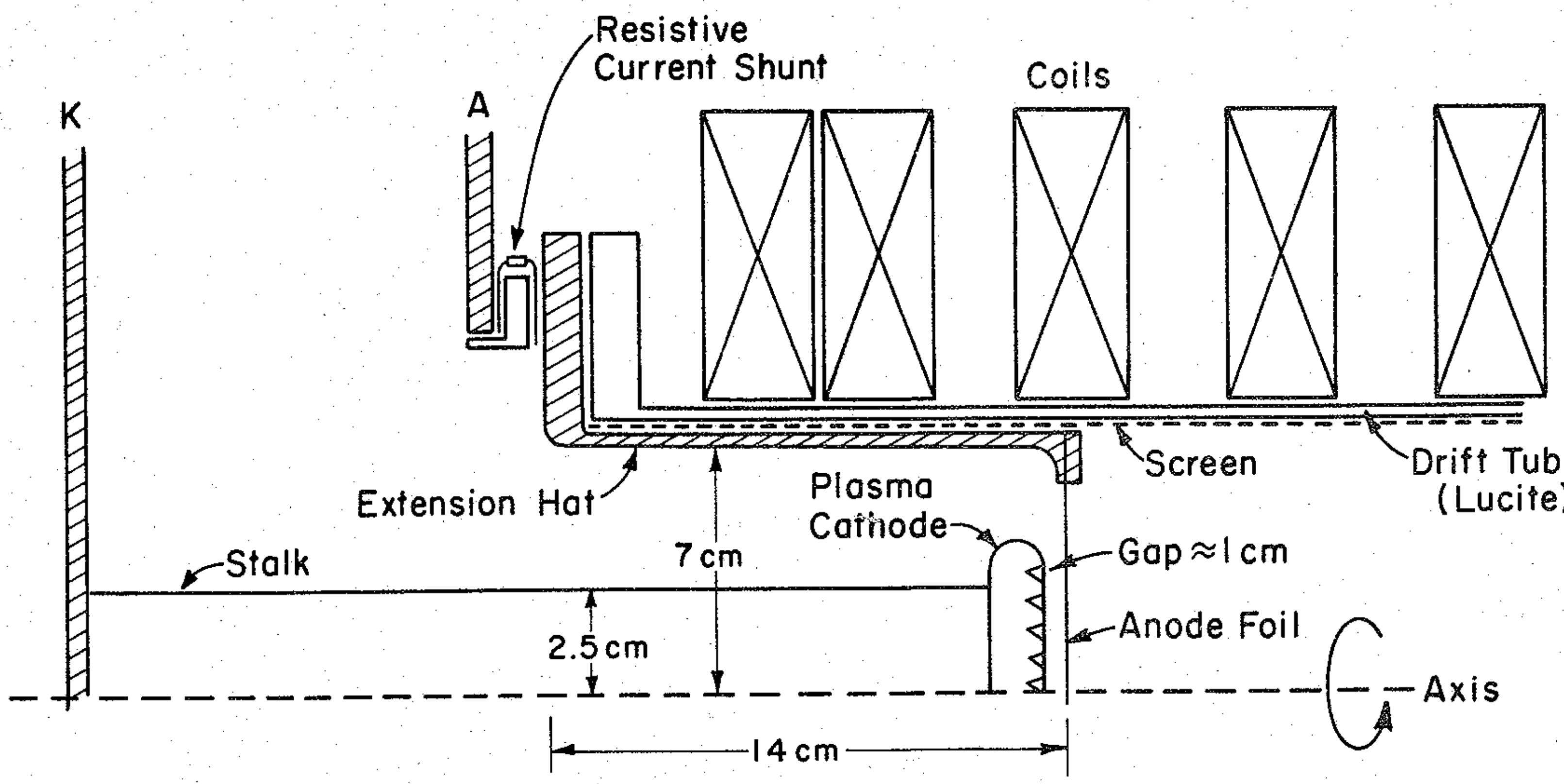


Fig. 3 Diagram of the anode-cathode gap extension into the magnetic field.

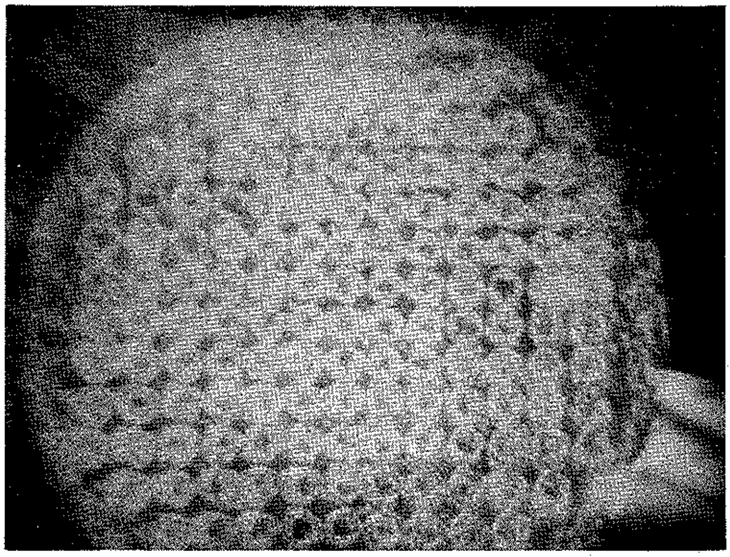
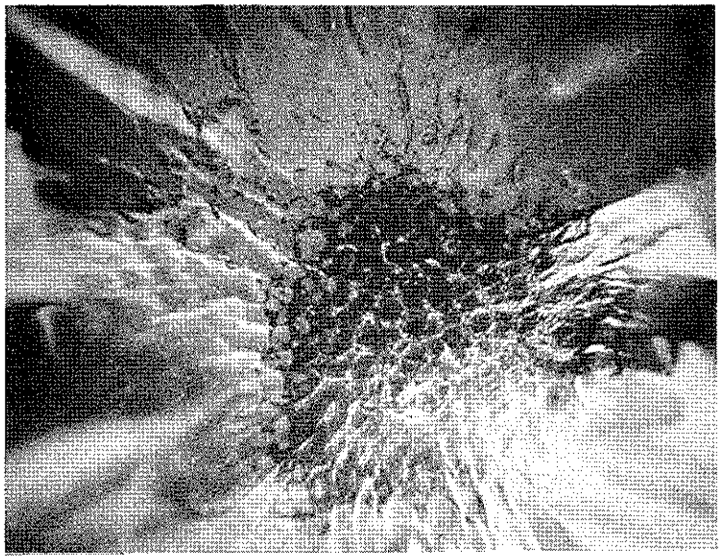
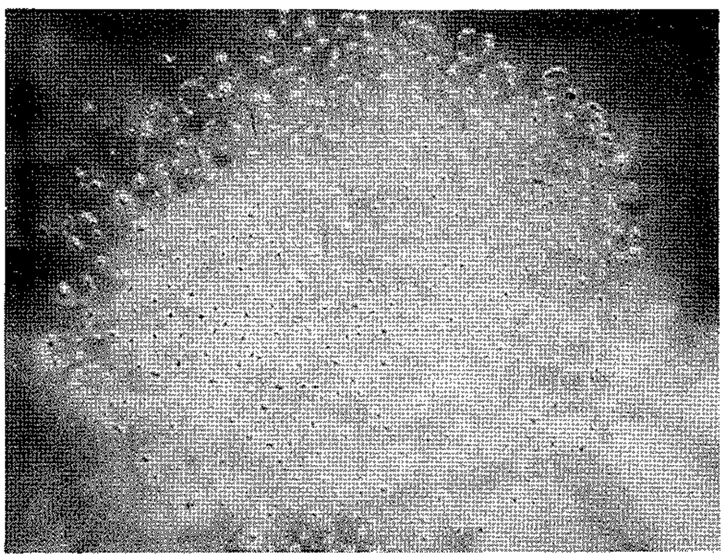


Fig. 4 Cathode image etched from damage on an anode screen.



(a.)



(b.)

Fig. 5 (a.) Foil damage without a magnetic field.
(b.) Foil damage with a 10 kG magnetic field.

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